Primordial power spectrum in effective theories of inflation

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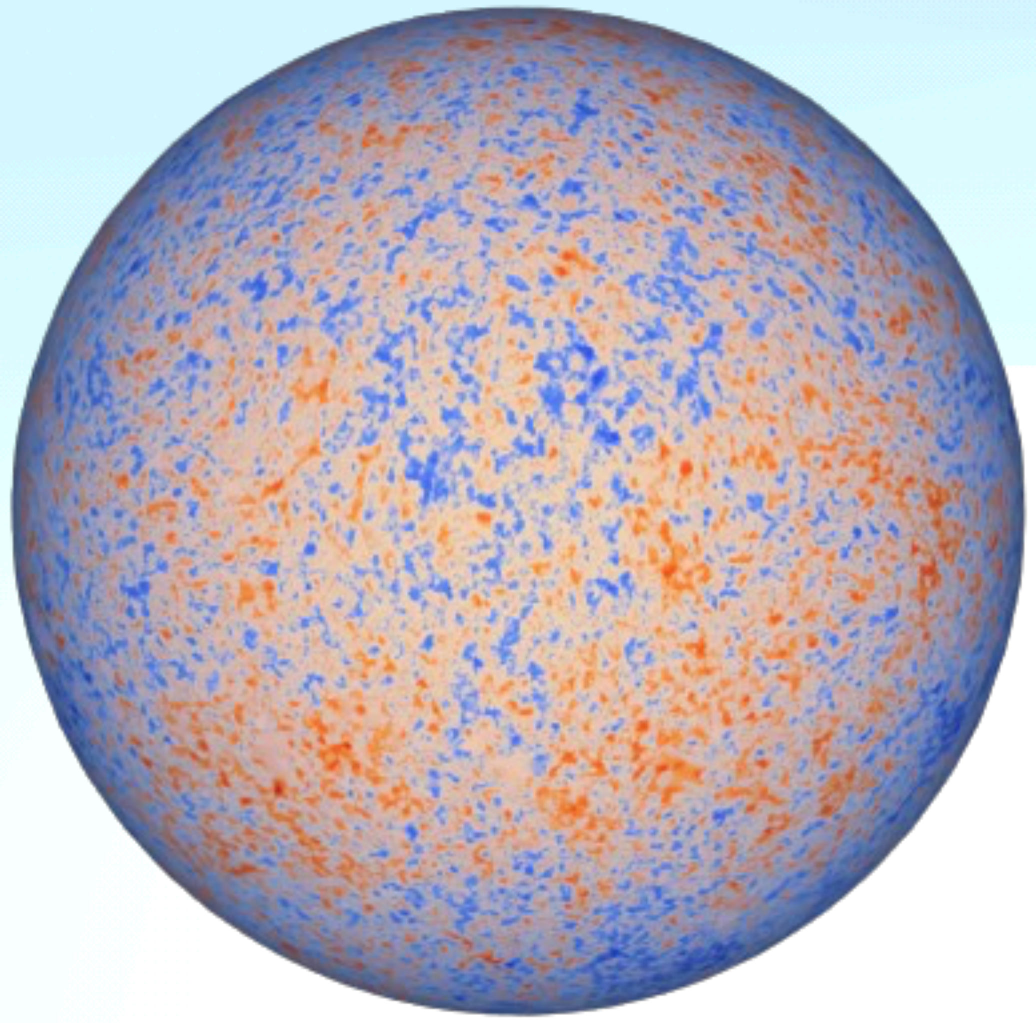
Based on work in collaboration with Eugenio Bianchi [arXiv: 2405.03157 and 2409:????]



International Loop Quantum Gravity Seminar — September 10, 2024

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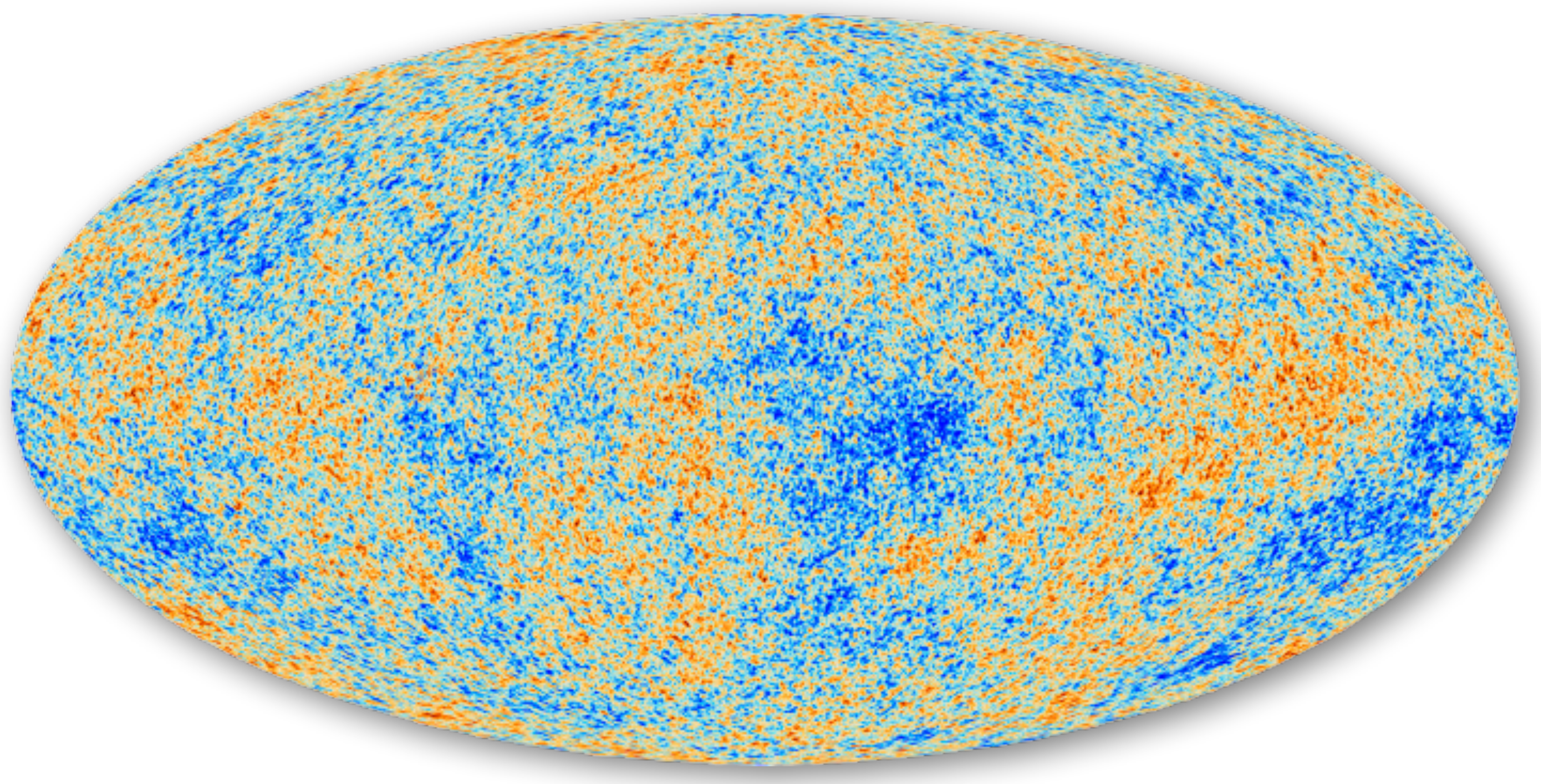
Current clues from nature

(Baumann, 2021)



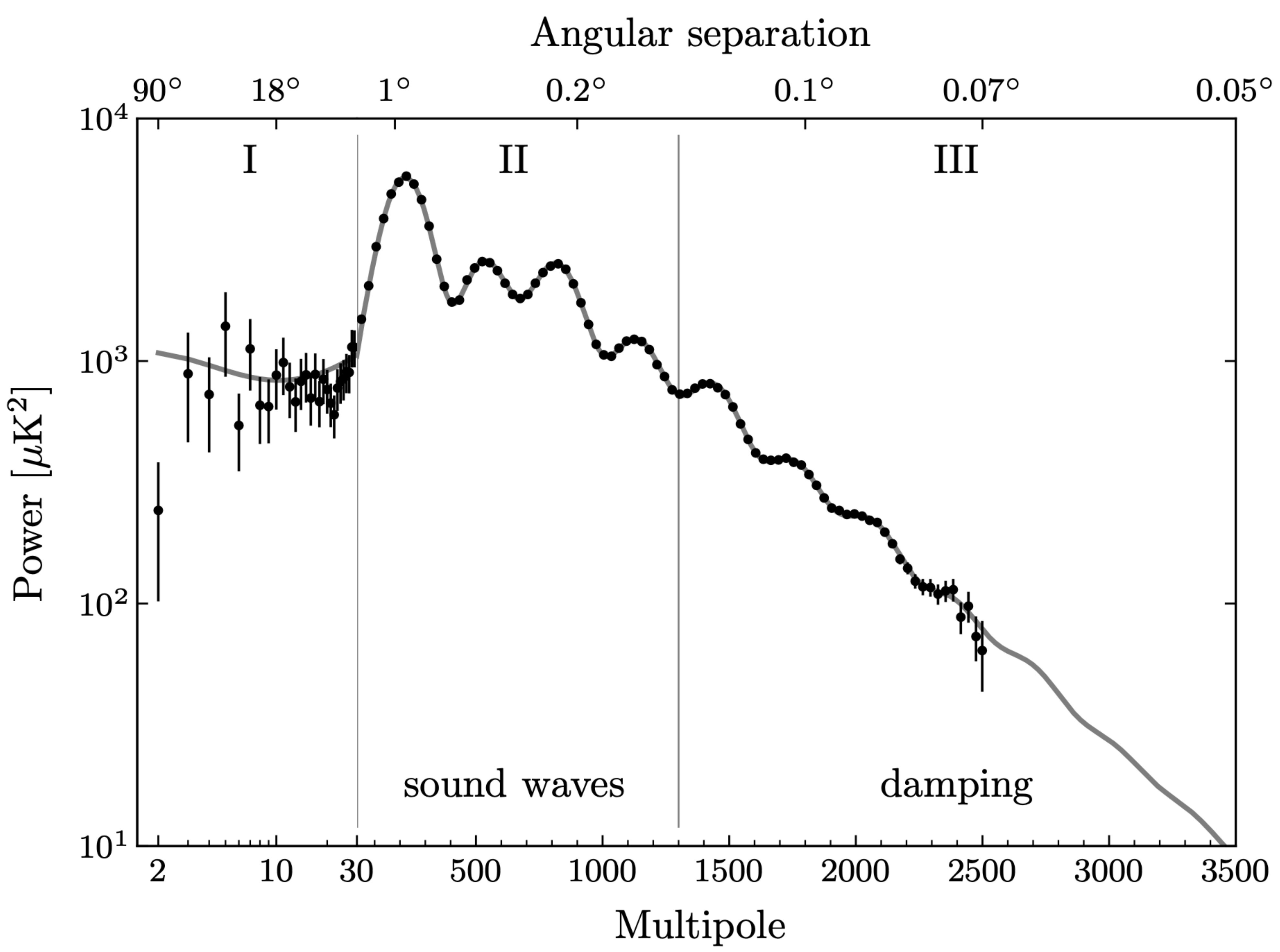
Temperature anisotropies of the CMB

Oldest direct observable of the Universe

(WMAP 2003, Planck 2018)

$$T(\mathbf{n}) = T_0\big(1 + \Theta(\mathbf{n})\big)$$

$$\langle \Theta(\mathbf{n})\Theta(\mathbf{n}')\rangle = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos(\theta))$$

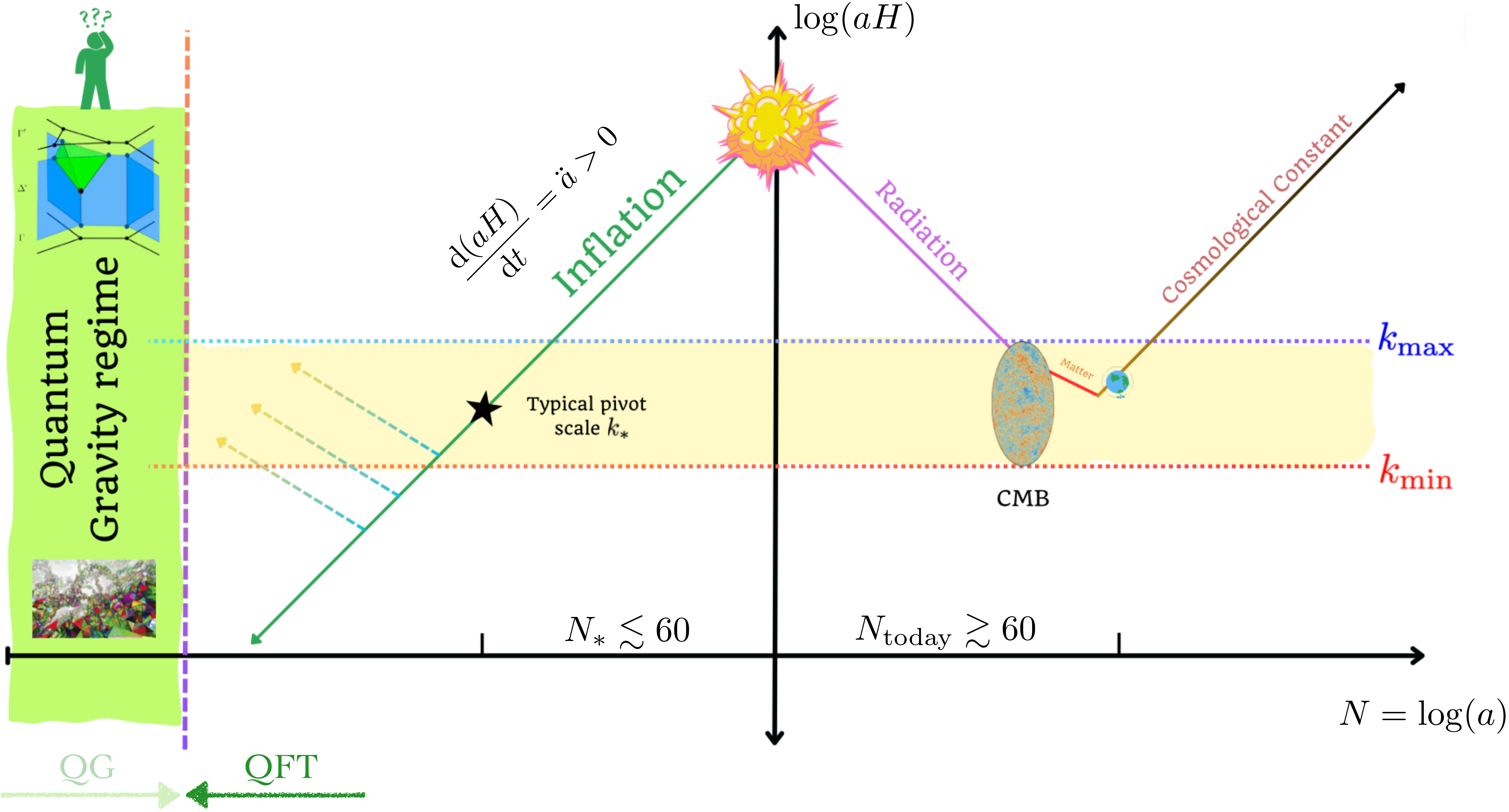


$$C_\ell = 4\pi \int \mathrm{d}\ln(k)\, \Theta_\ell^2(k) \mathcal{P}_{\mathcal{R}}(k)$$

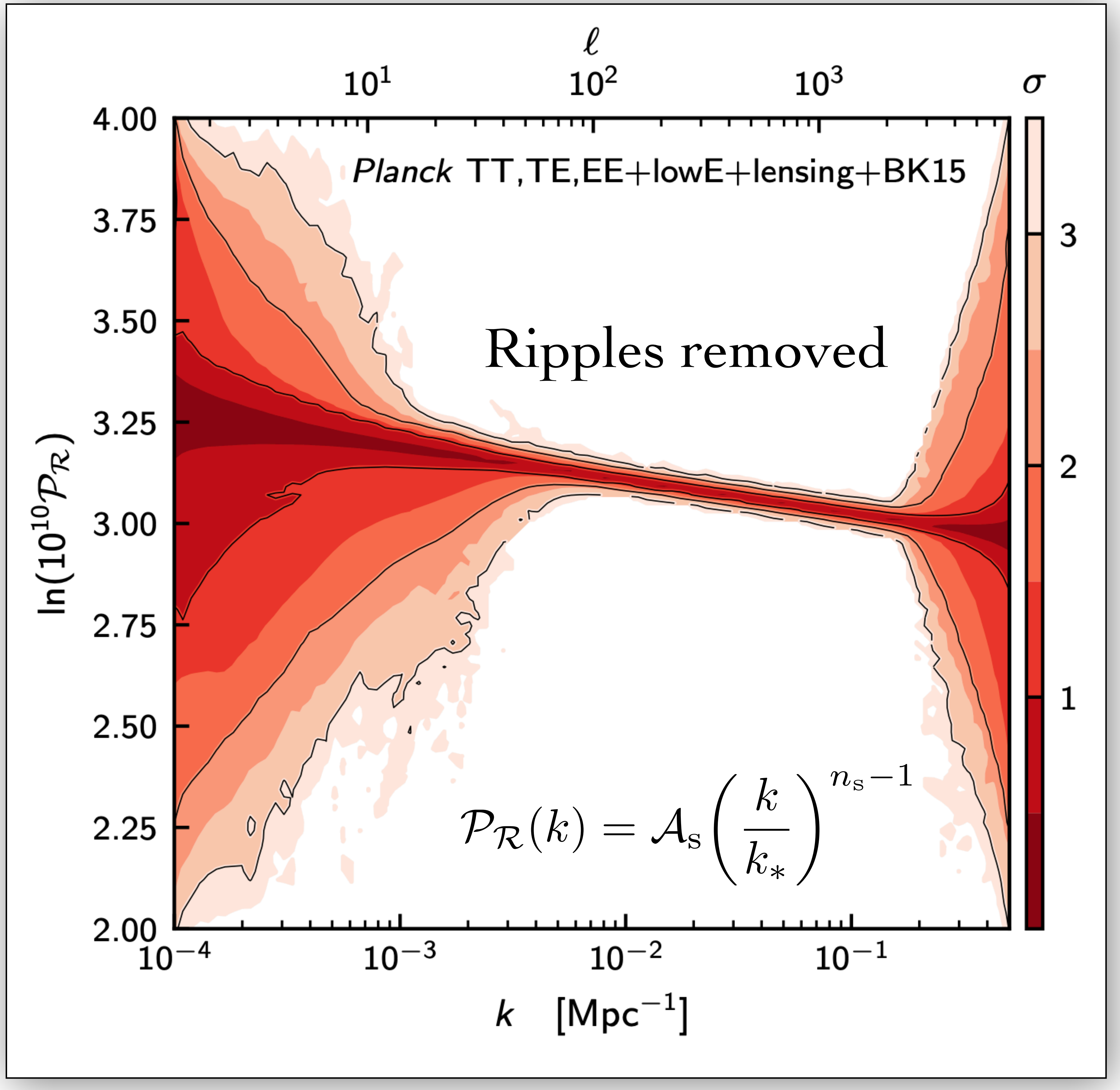
Transfer function

Primordial power spectrum of curvature fluctuations

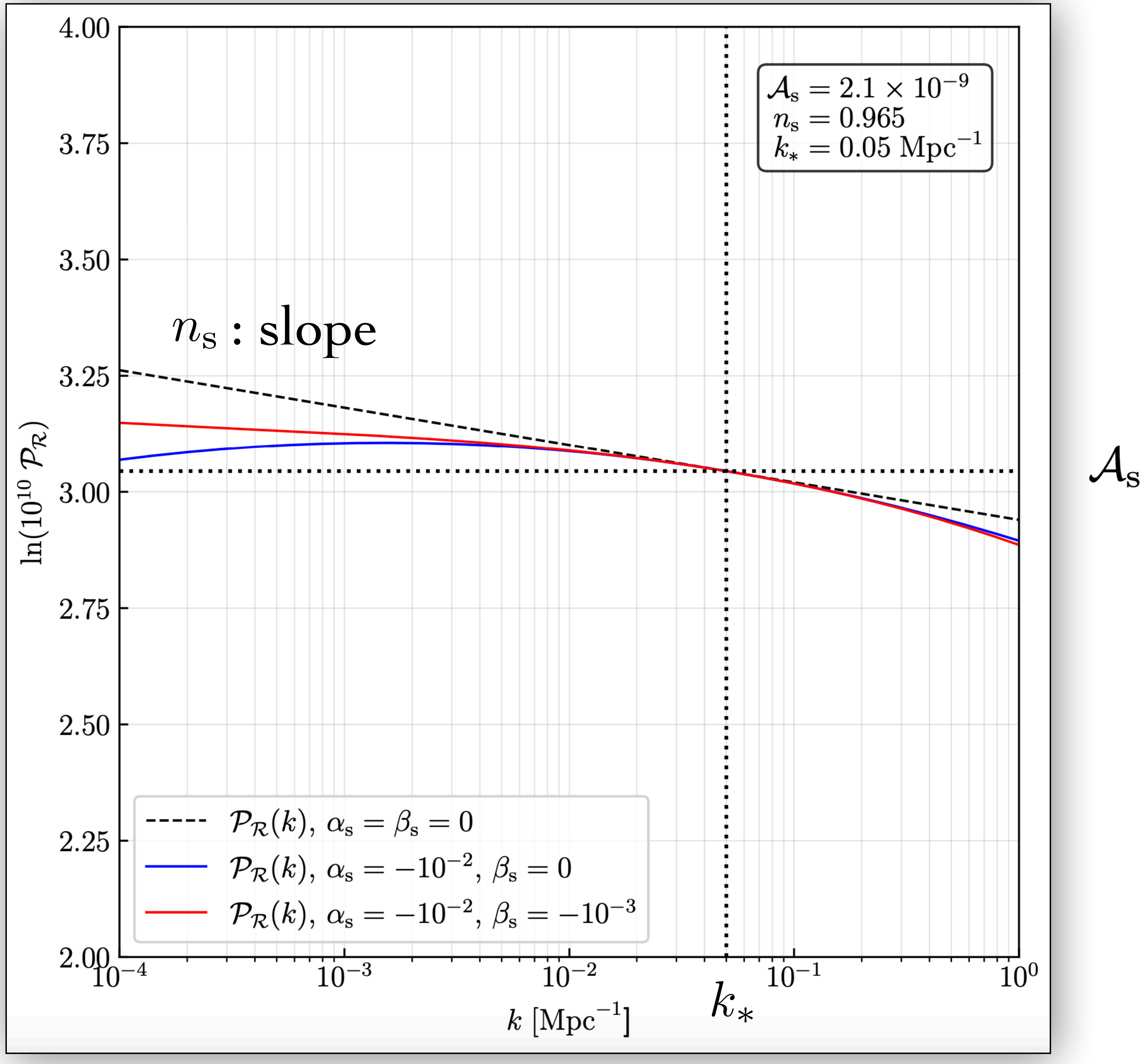
Window for QG phenomenology?



Qualitative features of the Power Spectrum



(Planck Collaboration, 2018)



α_s, β_s : 2nd and 3rd (log) derivatives
(exaggerated values)

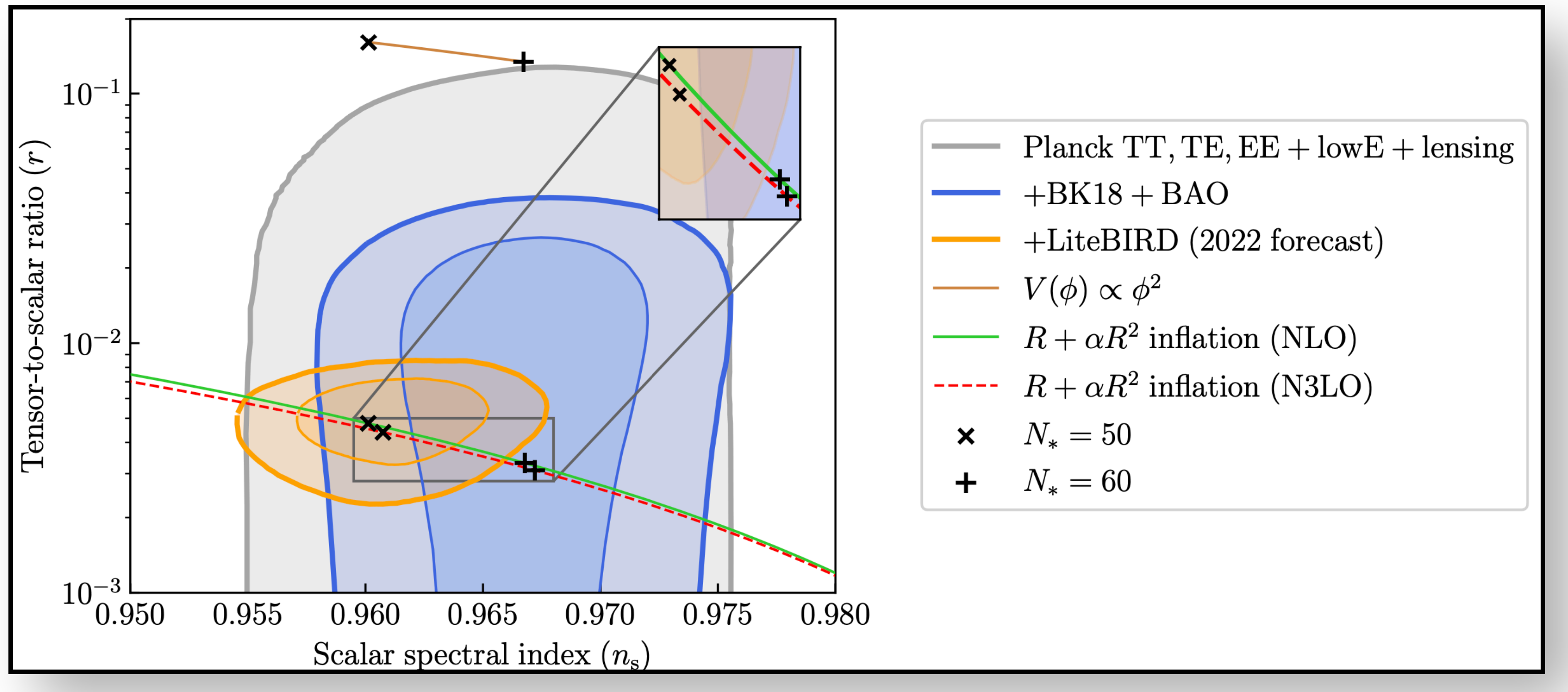
What did we work on?

$$\mathcal{P}_0^{(\psi)}(k) = \frac{\hbar H_*^2}{4\pi^2 c_{\psi*}^3 Z_{\psi*}}\left[1 + p_{0*} + p_{1*}\ln\left(\frac{k}{k_*}\right) + p_{2*}\ln\left(\frac{k}{k_*}\right)^2 + p_{3*}\ln\left(\frac{k}{k_*}\right)^3\right]$$

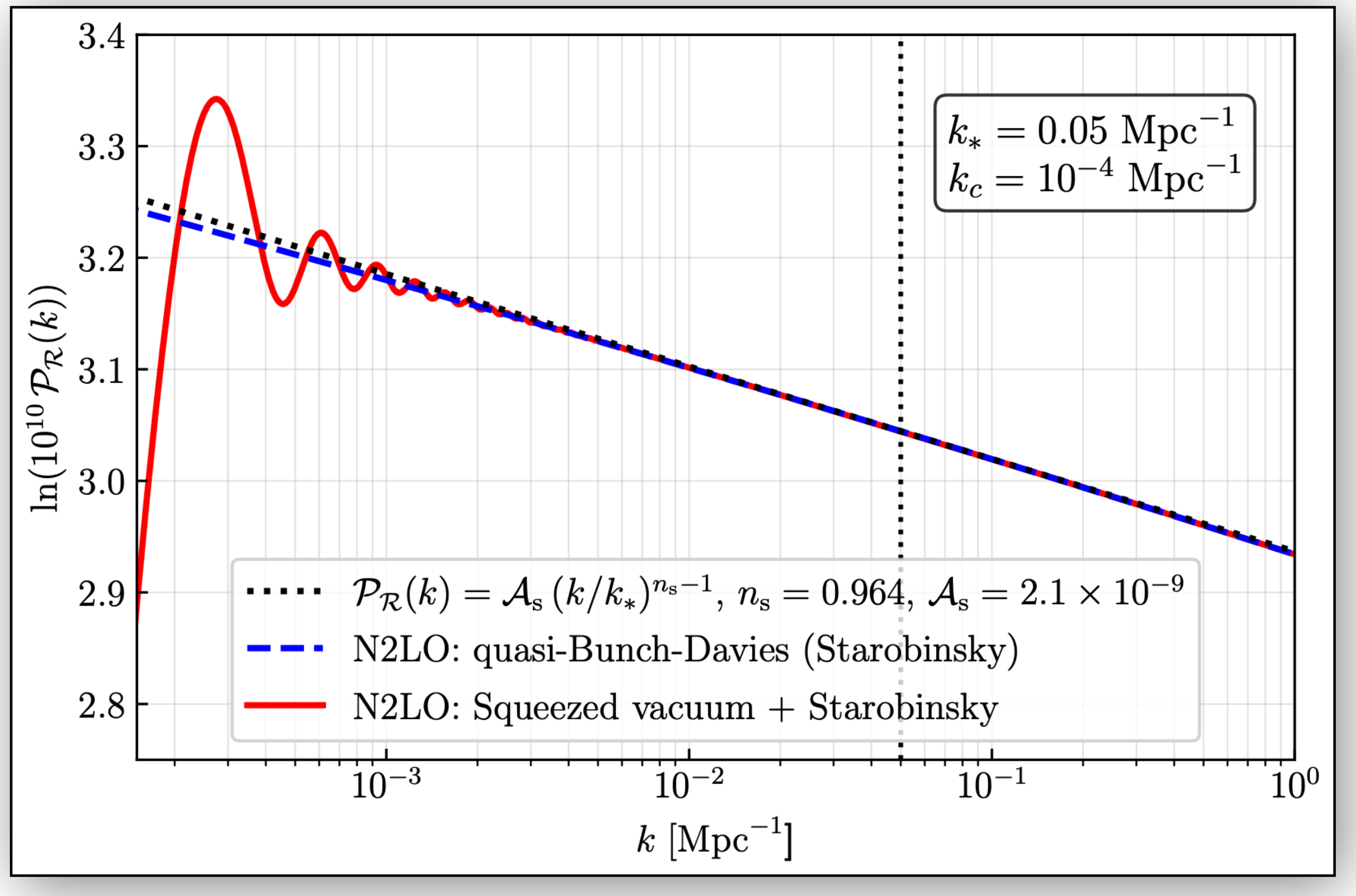
I. A general framework for the power spectrum at next-to-next-to-next-to leading order (N3LO)

II. Study purely gravitational models of inflation (e.g., Starobinsky, and any other effective theory)

III. Pre-inflationary epoch: Extension to a non-Bunch-Davies vacuum (e.g., squeezed vacua)



(Bianchi & Gamonal, 2024a)



(Bianchi & Gamonal, 2024b)

PART I: THE FRAMEWORK

Main ingredients

1. A gravitational action (matter optional, not required):

$$S = S[g_{\mu\nu}, \chi] \quad \text{with} \quad \begin{array}{r} g_{\mu\nu}(\mathbf{x},t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(\mathbf{x},t) \\ \chi(\mathbf{x},t) = \bar{\chi}(t) + \delta\chi(\mathbf{x},t) \ \text{(optional)} \end{array}$$

2. Small perturbations (on a flat FLRW background): Any dynamical SVT mode ($\Psi = \mathcal{R}, h_{\pm}, \ldots$)

$$\boxed{S^{(2)}_{\Psi}[\Psi] = \frac{1}{2}\int \mathrm{d}^4x \, \underbrace{Z_{\Psi}(t)}_{\text{Kinetic amplitude}} \, \underbrace{a(t)^3}_{\text{Scale factor}} \left(\dot{\Psi}^2 - \underbrace{\frac{c_{\Psi}(t)^2}{a(t)^2}}_{\text{Speed of sound}}(\partial_i \Psi)^2\right)}$$

Assumptions:

$$Z_{\Psi} = Z_{\Psi}(\cancel{k}, t), \; c_{\Psi} = c_{\Psi}(\cancel{k}, t)$$

$$Z_{\Psi} > 0, \; c_{\Psi}^2 \geq 0$$

c.f. single scalar field: $Z_{\mathrm{s}}(t) = \dfrac{\epsilon_{1H}(t)}{4\pi G}$, $\quad Z_{\mathrm{t}}(t) = \dfrac{1}{64\pi G}$, $\quad c_{\mathrm{t}}(t) = c_{\mathrm{s}}(t) = 1$

THE FRAMEWORK: N3LO FOR EFFECTIVE THEORIES OF INFLATION

Main ingredients

3. Hubble-flow expansion: Deviations from exact de Sitter parametrized by dimensionless ϵ's:

Hubble rate: $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$

Accelerated expansion: $\ddot{a}(t) = \left(1 - \epsilon_{1H}(t)\right) a(t) H(t)^2 > 0 \qquad \epsilon_{1H}(t) < 1$

During inflationary epoch: $\epsilon_{n\rho *} \ll 1$ Hubble-flow expansion at N3LO: $\mathcal{O}\left(\epsilon_*^3\right)$

$$\epsilon_{1H}(t) \equiv -\frac{\dot{H}(t)}{H(t)^2} \quad \epsilon_{1Z}(t) \equiv -\frac{\dot{Z}_\psi(t)}{H(t) Z_\psi(t)} \quad \epsilon_{1c}(t) \equiv -\frac{\dot{c}_\psi(t)}{H(t) c_\psi(t)} \quad \cdots \quad \epsilon_{(n+1)\rho}(t) \equiv -\frac{\dot{\epsilon}_{n\rho}(t)}{H(t)\, \epsilon_{n\rho}(t)}$$

Recall the quadratic action: $S_\Psi^{(2)}[\Psi] = \frac{1}{2} \int \mathrm{d}^4 x\, Z_\Psi(t)\, a(t)^3 \left(\dot{\Psi}^2 - \frac{c_\Psi(t)^2}{a(t)^2} (\partial_i \Psi)^2\right)$

A large class of effective theories of inflation

Theory	$Z_{\rm s}(t)$	$c_{\rm s}(t)$	$Z_{\rm t}(t)$	$c_{\rm t}(t)$
Single-field [22]	Eq. (74)	1	Eq. (75)	1
$R+\alpha R^2$ [23]	Eq. (96)	1	Eq. (99)	1
K-inflation [24]	✓	✓	✓	1
LQC+inflaton [25, 26]	✓	✓	✓	✓
$f(\varphi)$-Gauss Bonnet [27]	✓	✓	✓	✓
$f(\varphi)$-Chern Simons [28, 29]	✓	✓	×	×
General scalar-tensor [23]	✓	✓	✓	✓
Goldston mode EFT [20]	✓	✓	✓	✓
Multifield EFT [30]	✓	✓	✓	✓
Minimally broken CFT [31]	✓	✓	✓†	✓†
Weinberg's EFT [32]	✓	✓	✓†	✓†

Assumptions:

$$Z_\Psi = Z_\Psi(\cancel{k}, t),\; c_\Psi = c_\Psi(\cancel{k}, t)$$

$$Z_\Psi > 0,\; c_\Psi^2 \geq 0$$

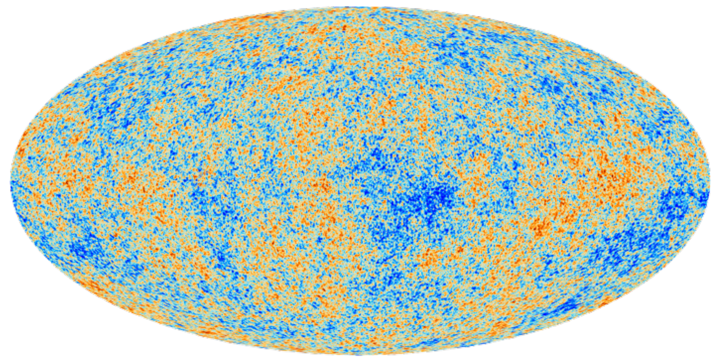
† : Framework works only if parity is preserved

$$S_\Psi^{(2)}[\Psi] = \frac{1}{2}\int \mathrm{d}^4x\, Z_\Psi(t)\, a(t)^3 \left(\dot{\Psi}^2 - \frac{c_\Psi(t)^2}{a(t)^2}(\partial_i\Psi)^2\right)$$

The Technique for the general case: Z_Ψ, c_Ψ arbitrary

How to find analytical expressions for two-point correlation functions?

(Mukhanov-Sasaki+time reparametrization)

Recall:  $\langle 0|\,\hat{\Psi}_f(t)\hat{\Psi}_f(t)\,|0\rangle = \int_0^\infty \frac{\mathrm{d}k}{k}\,\frac{k^3}{2\pi^2}\,|u(k,t)|^2\,\left|\tilde{f}(k)\right|^2 \longrightarrow u(k,t(y)) \to \frac{y\,w(y)}{\sqrt{2\,k^3\,\mu(y)}}$

New time variable: $y = -k\tau = \frac{k\,\tilde{c}}{aH}$, expansion of $\epsilon_{1H}(t) \to \epsilon_{1H}(y) = \epsilon_{1Hk} + (\ldots) \times \log(y/y_k)$, and so on, around $y_k = 1$:

$$\boxed{w''(y) + \left(1 - \frac{2}{y^2}\right) w(y) = \frac{g(y)}{y^2}\,w(y)\,, \qquad g(y) = g_{1k} + g_{2k}\ln(y) + g_{3k}\ln(y)^2 + \cdots}$$

Leading order: $w''(y) + \left(1 - \frac{2}{y^2}\right) w(y) = 0 \longrightarrow w(y) = \left(1 + \frac{i}{y}\right) e^{iy}$ (Bunch-Davies)

NLO: $w''(y) + \left(1 - \frac{2 + g_{1k}}{y^2}\right) w(y) = 0 \longrightarrow w(y) = \sqrt{\frac{\pi\,y}{2}}\,(J_\nu(y) - iY_\nu(y))$

N2LO: $w''(y) + \left(1 - \frac{2 + g_{1k} + g_{2k}\ln(y)}{y^2}\right) w(y) = 0 \longrightarrow w(y) = ???$

The Technique: Green's function method

A systematic expansion computable order-by-order around Bunch-Davies

[Gong & Stewart '01; Auclair & Ringeval '22]

Mode eq: $$w''(y) + \left(1 - \frac{2}{y^2}\right) w(y) = \frac{g(y)}{y^2}\, w(y)$$

Wronskian: $$w(y) w'^*(y) - w'(y) w^*(y) = -2\,\mathrm{i}$$

$$\begin{cases} G(y,s) = \frac{\mathrm{i}}{2}\left(w_0(y) w_0^*(s) - w_0(s) w_0^*(y)\right)\Theta(s-y) \\ w_0(y) = \left(1 + \frac{\mathrm{i}}{y}\right) \mathrm{e}^{\mathrm{i}y} \end{cases}$$

[Bunch & Davies '78]

Quasi-Bunch-Davies vacuum state

$$\boxed{\begin{aligned} w_{\mathrm{qBD}}(y) &= w_0(y) + \int_y^{\infty} \frac{g(y)}{s^2}\, w(s)\, G(y,s)\ \mathrm{d}s \\ &= w_0(y) + w_1(y) + w_2(y) + w_3(y) + \mathcal{O}\left(\epsilon^4\right) \end{aligned}}$$

Late-time power spectrum

$$\begin{aligned} \mathcal{P}_{\mathrm{qBD}}(k) &\equiv \lim_{t\to\infty} \frac{k^3}{2\pi^2} \left|u_{\mathrm{qBD}}(k,t)\right|^2 \\ &= \lim_{y\to 0^+} \frac{\left|y\, w_{\mathrm{qBD}}(y)\right|^2}{4\pi^2\, \mu(y)} \end{aligned}$$

The details of the physics is encoded in $g(y) = g_{1k} + g_{2k}\ln(y) + g_{3k}\ln(y)^2 + \cdots$

The results: Primordial Power Spectrum at N3LO

Evaluating around pivot scale $y_* = k/k_*$, $\epsilon_{1Hk} \to \epsilon(y_k/y_*) = \epsilon_{1H*} - (\ldots)\ \ln(k/k_*) + \cdots$

$$\mathcal{P}_0^{(\psi)}(k) = \frac{\hbar H_*^2}{4\pi^2 c_{\psi *}^3 Z_{\psi *}}\left[1 + \underbrace{p_{0*} + p_{1*}}_{\mathcal{O}(\epsilon_*)} \ln\left(\frac{k}{k_*}\right) + \underbrace{p_{2*}}_{\mathcal{O}(\epsilon_*^2)} \ln\left(\frac{k}{k_*}\right)^2 + \underbrace{p_{3*}}_{\mathcal{O}(\epsilon_*^3)} \ln\left(\frac{k}{k_*}\right)^3\right]$$

starts at: $\mathcal{O}(\epsilon_*)$ $\qquad$ $\mathcal{O}(\epsilon_*^2)$ $\qquad$ $\mathcal{O}(\epsilon_*^3)$

Amplitude: $\mathcal{A}_* \equiv \mathcal{P}_0(k_*)$

Tilt: $\theta_* \equiv k\dfrac{\mathrm{d}}{\mathrm{d}k}\ln(\mathcal{P}_0(k))\Big|_{k=k_*}$, $\quad (n_\mathrm{s} \equiv 1 + \theta_\mathrm{s} \quad n_\mathrm{t} \equiv \theta_\mathrm{t})$

Running of the tilt: $\alpha_* \equiv k\dfrac{\mathrm{d}}{\mathrm{d}k}\left[k\dfrac{\mathrm{d}}{\mathrm{d}k}\ln(\mathcal{P}_0(k))\right]\Big|_{k=k_*}$

Running of the running of the tilt: $\beta_* \equiv k\dfrac{\mathrm{d}}{\mathrm{d}k}\left\{k\dfrac{\mathrm{d}}{\mathrm{d}k}\left[k\dfrac{\mathrm{d}}{\mathrm{d}k}\ln(\mathcal{P}_0(k))\right]\right\}\Big|_{k=k_*}$

Quantity	Order	Expression [Bianchi & MG 2024a]
$\theta_*^{(\psi)}$	NLO :	$-2\epsilon_{1H*}+\epsilon_{1Z*}+3\epsilon_{1c*}$
	N2LO :	$-2\epsilon_{1H*}^2+2(1+C)\epsilon_{1H*}\epsilon_{2H*}+\epsilon_{1Z*}(\epsilon_{1H*}-C\epsilon_{2Z*})+\epsilon_{1c*}(5\epsilon_{1H*}-3\epsilon_{1c*}-\epsilon_{1Z*})-(2+3C)\epsilon_{1c*}\epsilon_{2c*}$
	N3LO :	$-2\epsilon_{1H*}^3+(14+6C-\pi^2)\epsilon_{1H*}^2\epsilon_{2H*}+\frac{1}{12}(-24-24C-12C^2+\pi^2)\epsilon_{1H*}\epsilon_{2H*}^2$ $+\frac{1}{12}(-24-24C-12C^2+\pi^2)\epsilon_{1H*}\epsilon_{2H*}\epsilon_{3H*}+\epsilon_{1H*}^2\epsilon_{1Z*}+\frac{1}{2}(-10-2C+\pi^2)\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2H*}$ $+\frac{1}{2}(-8-4C+\pi^2)\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2Z*}+\frac{1}{4}(8-\pi^2)\epsilon_{1Z*}^2\epsilon_{2Z*}+\frac{1}{24}(12C^2-\pi^2)\epsilon_{1Z*}\epsilon_{2Z*}^2$ $+\frac{1}{24}(12C^2-\pi^2)\epsilon_{1Z*}\epsilon_{2Z*}\epsilon_{3Z*}+3\epsilon_{1c*}^3-8\epsilon_{1c*}^2\epsilon_{1H*}+7\epsilon_{1c*}\epsilon_{1H*}^2+\epsilon_{1c*}^2\epsilon_{1Z*}-2\epsilon_{1c*}\epsilon_{1H*}\epsilon_{1Z*}$ $+\frac{1}{4}(100+36C-9\pi^2)\epsilon_{1c*}^2\epsilon_{2c*}+\frac{1}{2}(-36-16C+3\pi^2)\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2c*}+\frac{1}{4}(28+4C-3\pi^2)\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2c*}$ $+\frac{1}{8}(16+16C+12C^2-\pi^2)\epsilon_{1c*}\epsilon_{2c*}^2+\frac{1}{2}(-38-14C+3\pi^2)\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2H*}$ $+\frac{1}{4}(24+8C-3\pi^2)\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2Z*}+\frac{1}{8}(16+16C+12C^2-\pi^2)\epsilon_{1c*}\epsilon_{2c*}\epsilon_{3c*}$
$\alpha_*^{(\psi)}$	N2LO :	$2\epsilon_{1H*}\epsilon_{2H*}-\epsilon_{1Z*}\epsilon_{2Z*}-3\epsilon_{1c*}\epsilon_{2c*}$
	N3LO :	$+6\epsilon_{1H*}^2\epsilon_{2H*}-2(1+C)\epsilon_{1H*}\epsilon_{2H*}^2-2(1+C)\epsilon_{1H*}\epsilon_{2H*}\epsilon_{3H*}-\epsilon_{1H*}\epsilon_{2H*}\epsilon_{1Z*}-2\epsilon_{1H*}\epsilon_{1Z*}\epsilon_{2Z*}$ $+C\epsilon_{1Z*}\epsilon_{2Z*}^2+C\epsilon_{1Z*}\epsilon_{2Z*}\epsilon_{3Z*}+9\epsilon_{1c*}^2\epsilon_{2c*}-8\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2c*}+\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2c*}+(2+3C)\epsilon_{1c*}\epsilon_{2c*}^2$ $-7\epsilon_{1c*}\epsilon_{1H*}\epsilon_{2H*}+2\epsilon_{1c*}\epsilon_{1Z*}\epsilon_{2Z*}+(2+3C)\epsilon_{1c*}\epsilon_{2c*}\epsilon_{3c*}$
$\beta_*^{(\psi)}$	N3LO :	$-2\epsilon_{1H*}\epsilon_{2H*}(\epsilon_{2H*}+\epsilon_{3H*})+\epsilon_{1Z*}\epsilon_{2Z*}(\epsilon_{2Z*}+\epsilon_{3Z*})+3\epsilon_{1c*}\epsilon_{2c*}(\epsilon_{2c*}+\epsilon_{3c*})$

Part II: Starobinsky inflation

Can we describe the inflationary epoch as a purely gravitational phenomenon?

What is the simplest model exhibiting this characteristic
that remains consistent with observational constraints?

Starobinsky inflation in the geometric frame

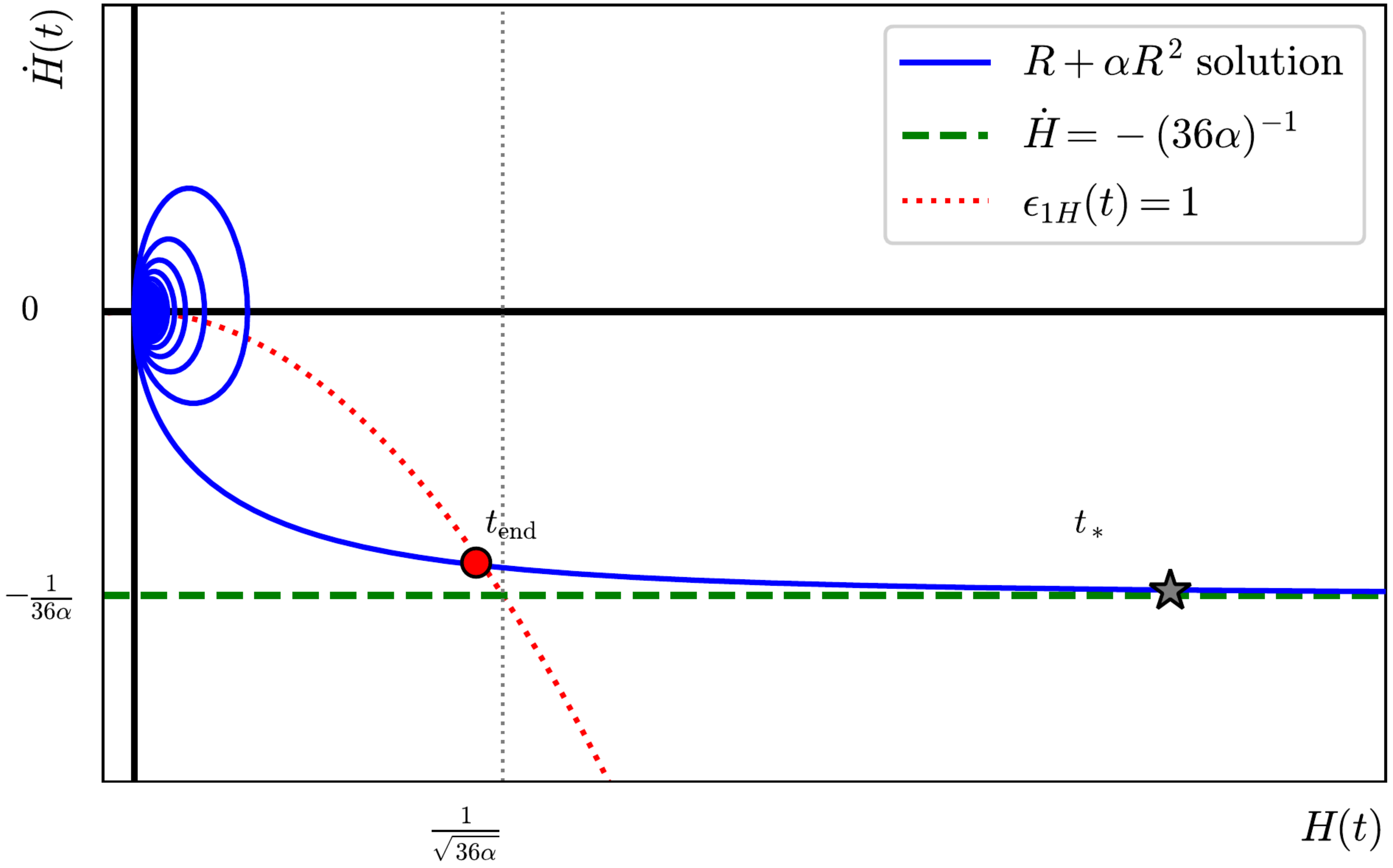
[Starobinsky '79-'80; Vilenkin '85; De Felice & Tsujikawa '10]

Not a fundamental theory
but a sector of semi-classical gravity/EFT

$$\boxed{S[g_{\mu\nu}] = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\,(R+\alpha R^2)} \longrightarrow$$

Modified Friedmann equation (background):

$$H(t)^2 + 6\alpha H(t)^4\epsilon_{1H}(t)\Big(3\epsilon_{1H}(t) + 2\epsilon_{2H}(t) - 6\Big) = 0$$



1. **Purely geometrical** (dependance on H, and time derivatives). No scalar field.

2. Physical scale: $\alpha \sim 10^{10}\,\ell_P^2$

3. Approximate behavior nearby t_*:

$$a(t) \sim a_*\, e^{H_*(t-t_*) - \frac{1}{72\alpha}(t-t_*)^2}$$

$$R \sim 12H(t)^2 - \frac{1}{6\alpha}$$

Starobinsky inflation in the geometric frame

[Starobinsky '79-'80; Vilenkin '85; De Felice & Tsujikawa '10]

$$\boxed{S[g_{\mu\nu}] = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\,(R+\alpha R^2)}$$

$\longrightarrow$

Only $g_{\mu\nu}(\mathbf{x},t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(\mathbf{x},t)$

3 d.o.f.s: 2 tensor polarizations + 1 scalar

No conformal transformation, physical metric.

From quadratic action of cosmological perturbations ($c_s = c_t = 1$):

$$\boxed{Z_\mathrm{s}(t) = \frac{3\bar{\chi}(t)}{16\pi G_\mathrm{N}}\left(\frac{\epsilon_\chi(t)}{1+\frac{1}{2}\epsilon_\chi(t)}\right)^2,\ Z_\mathrm{t}(t) = \frac{\bar{\chi}(t)}{64\pi G_\mathrm{N}}}$$

with

$$\bar{\chi}(t) = 1 + 2\alpha\bar{R} = 1 - 12\alpha H(t)^2(\epsilon_{1H}(t) - 2)$$

$$\epsilon_\chi(t) = -\frac{\dot{\bar{\chi}}(t)}{H(t)\bar{\chi}}$$

Simple to compute at leading order (LO), even at NLO.
Highly non-trivial at N2LO and beyond.

Results for Starobinsky inflation in the geometric frame

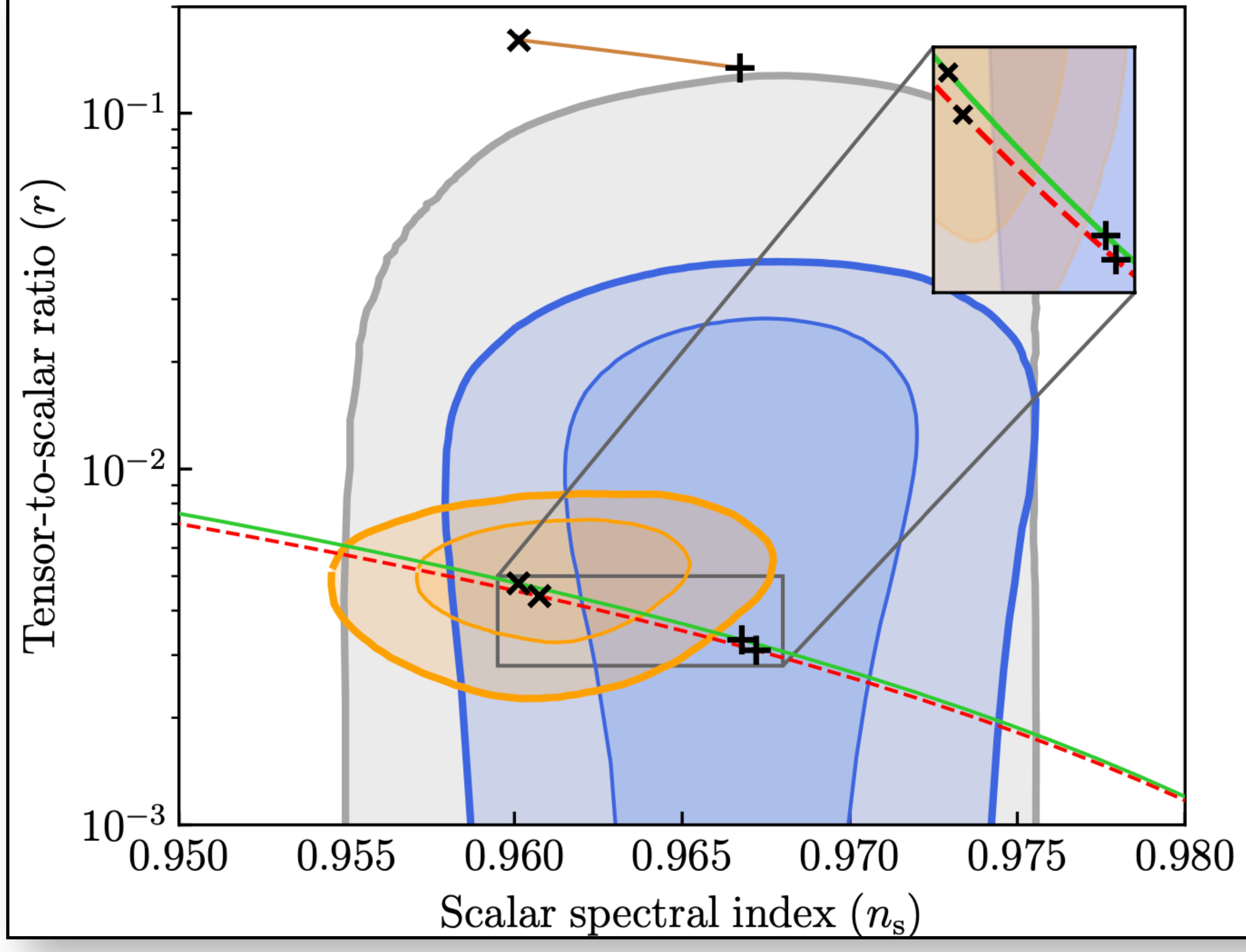
Amplitudes of the primordial power spectrum:

$$\mathcal{A}_{\mathrm{t}} = \frac{2G\hbar}{3\pi\alpha}(1+\cdots) \qquad \mathcal{A}_{\mathrm{s}} = \frac{G\hbar\, N_*^2}{18\pi\alpha}(1+\cdots)$$

$$\alpha = (2.777 \pm 0.779) \times 10^{10}\ \ell_P^2$$

7% decrease in tensor-to-scalar ratio:

$$\boxed{r = 3\,(n_{\mathrm{s}}-1)^2 + \tfrac{7}{2}(n_{\mathrm{s}}-1)^3 + \mathcal{O}\big((n_{\mathrm{s}}-1)^4\big)}$$



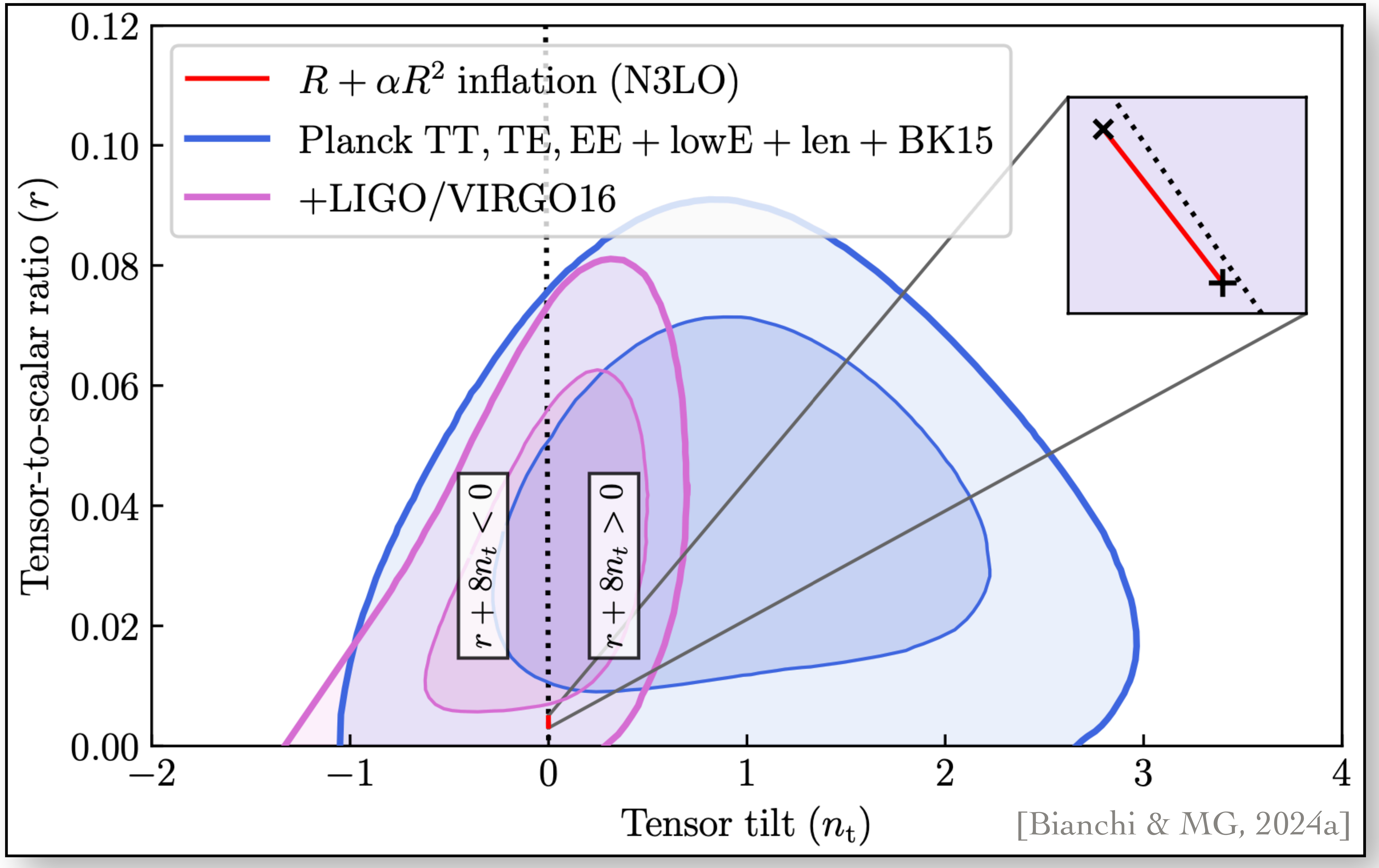
$$r = (3.767 \pm 0.652) \times 10^{-3} \quad \text{at N3LO}$$

[Bianchi & MG, 2024a]

Results for Starobinsky inflation in the geometric frame

Robust prediction: Small deviation from exact consistency condition

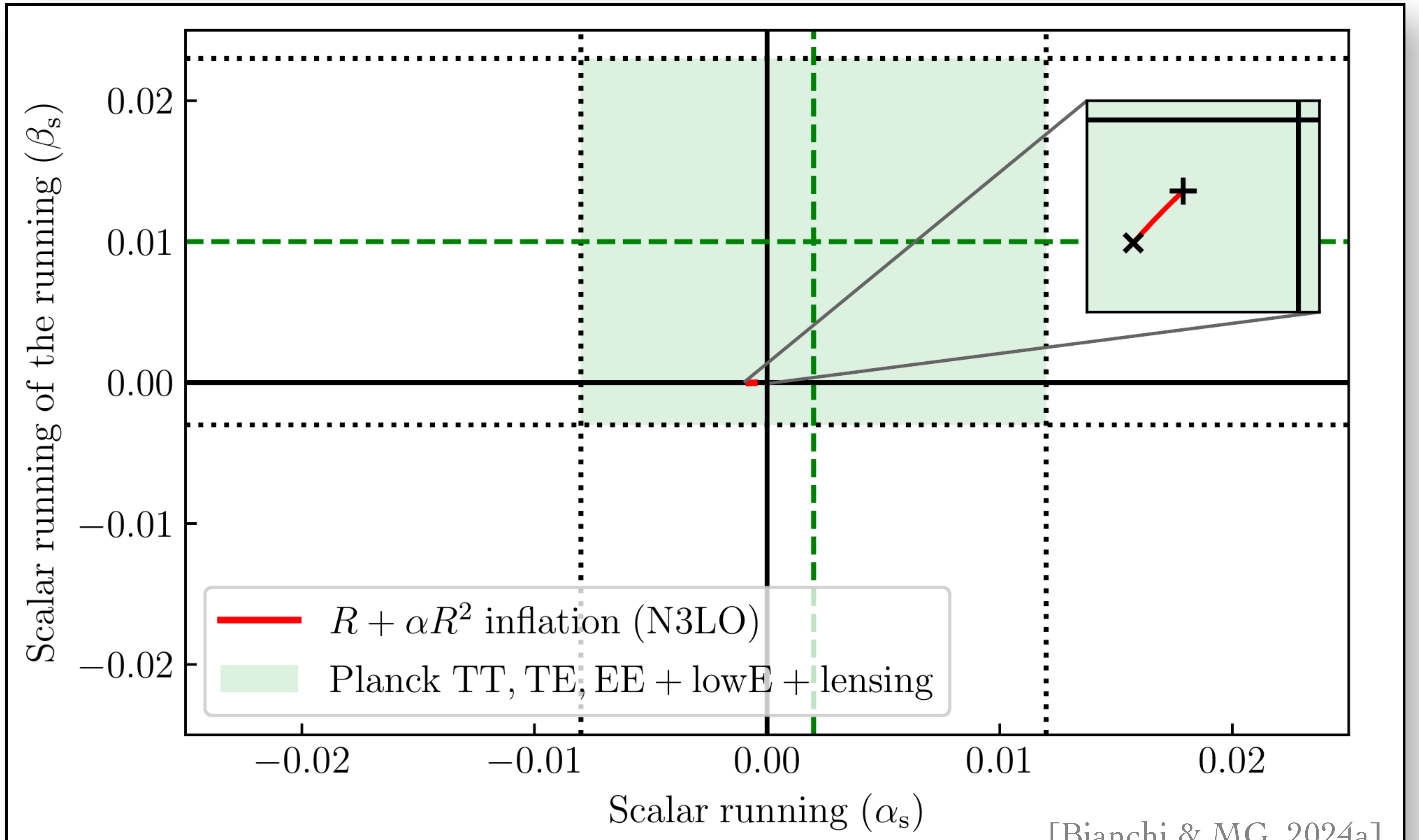
$$\boxed{r + 8n_{\rm t} = 6(n_{\rm s} - 1)^3 + \mathcal{O}\big((n_{\rm s} - 1)^4\big) \; = (-3.031 \pm 0.809) \times 10^{-4}} \quad \text{for } N_* = 55 \pm 5 \quad \text{at N3LO}$$



Results for Starobinsky inflation in the geometric frame

Robust prediction: Scalar running small and **negative**

$$\boxed{\alpha_\mathrm{s} = -\frac{1}{2}(n_\mathrm{s}-1)^2 + \frac{5}{48}(n_\mathrm{s}-1)^3 + \mathcal{O}\big((n_\mathrm{s}-1)^4\big)}$$



[Bianchi & MG, 2024a]

$$\alpha_\mathrm{s} = (-6.626 \pm 1.180) \times 10^{-4} \quad \text{at N3LO}$$
$$\beta_\mathrm{s} = (-2.526 \pm 0.674) \times 10^{-5} \quad \text{at N3LO}$$

for $N_* = 55 \pm 5$

(Don't confuse α_s, the running of the scalar tilt, with the Starobinsky coupling α)

What if the initial conditions were not exactly quasi-Bunch-Davies?

Past and future efforts to identify features in the power spectrum

In these scenarios, the primordial spectrum can generally be analytically approximated by an expression of the form

$$\ln \mathcal{P}_{\mathcal{R}}^{Y}(k) = \ln \mathcal{P}_{\mathcal{R}}^{0}(k) + \ln \Upsilon_Y(k/k_Y^{c}), \quad (63)$$

with $Y \in \{\mathrm{kin}, \mathrm{rad}, \mathrm{kink}\}$, where Υ_Y is a function with $\ln \Upsilon_Y \to 0$ in the limit $k \gg k_Y^{c}$ that describes the shape of the cutoff and the transition to a power-law spectrum at smaller scales.

(Planck 2018 results. X. Constraints on inflation)

4.1. Models

We consider a superimposed pattern of oscillations as

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R},0}(k)\left[1 + \delta P^{\mathrm{X}}(k)\right], \quad (27)$$

where $\mathcal{P}_{\mathcal{R},0}(k)$ is the standard power-law PPS of the comoving curvature perturbations $\mathcal{R}$ on superhorizon scales, given in Eq. (1). We study the following templates with superimposed oscillations on the PPS

$$\delta P^{\mathrm{X}}(k) = \mathcal{A}_{\mathrm{X}} \sin\left(\omega_{\mathrm{X}} \Xi_{\mathrm{X}} + 2\pi\phi_{\mathrm{X}}\right), \quad (28)$$

(Ballardini et al. "Euclid: The search for primordial features", 2024)

A scenario that has been explored in detail in the LQC community

Part III: Pre-inflationary epoch and Squeezed vacua

If $w(y)$ is a basis of solutions, after a Bogoliubov transformation in the Fock representation, the mode function

$$\tilde{w}(y) = \alpha_k\, w(y) + \beta_k\, w(y)^*$$

also represents a vacuum state, a "squeezed state", if it preserves the Wronskian condition:

$$\tilde{w}(y)\tilde{w}'^*(y) - \tilde{w}'(y)\tilde{w}^*(y) = -2\,\mathrm{i}\left(|\alpha_k|^2 - |\beta_k|^2\right) \;\rightarrow\; |\alpha_k|^2 - |\beta_k|^2 = 1$$

The standard expression in the literature for the power spectrum at leading order is

$$\mathcal{P}_{\mathrm{sqz}}^{(\mathrm{LO})}(k) \;=\; \left|\alpha_k - \beta_k\right|^2 \mathcal{P}_{\mathrm{qBD}}^{(\mathrm{LO})}(k)$$

Is it still true at NLO? N2LO?

Pre-inflationary epoch and Squeezed vacua

Note that: $$\Upsilon(k) \equiv \frac{\mathcal{P}_{\rm sqz}(k)}{\mathcal{P}_{\rm qBD}(k)} = \lim_{y\to 0^+} \frac{|\alpha_k\, y\, w(y) + \beta_k\, y\, w^*(y)|^2}{|y\, w(y)|^2}$$

$$= \lim_{y\to 0^+} \left| \alpha_k + \beta_k \frac{w^*(y)}{w(y)} \right|^2$$

$$\boxed{\Upsilon(k) = \left|\alpha_k - \beta_k\, \mathrm{e}^{\mathrm{i}\delta_k}\right|^2}$$

Generic feature, present at all orders

[Bianchi & MG, 2024b]

with $$\delta_k^{(\rm N2LO)} = -\frac{\pi}{3} g_{1k} + \frac{\pi}{27}\left(g_{1k}^2 + (9\,C-3)\, g_{2k}\right) + \mathcal{O}(\epsilon^3)$$

$$\sim \frac{\pi}{2}\left[(n_{\rm s}-1) + \left(C - \frac{1}{3}\right)\alpha_{\rm s} + \frac{1}{6}(n_{\rm s}-1)^2 + \cdots\right]$$

Leading contributions for curvature perturbations

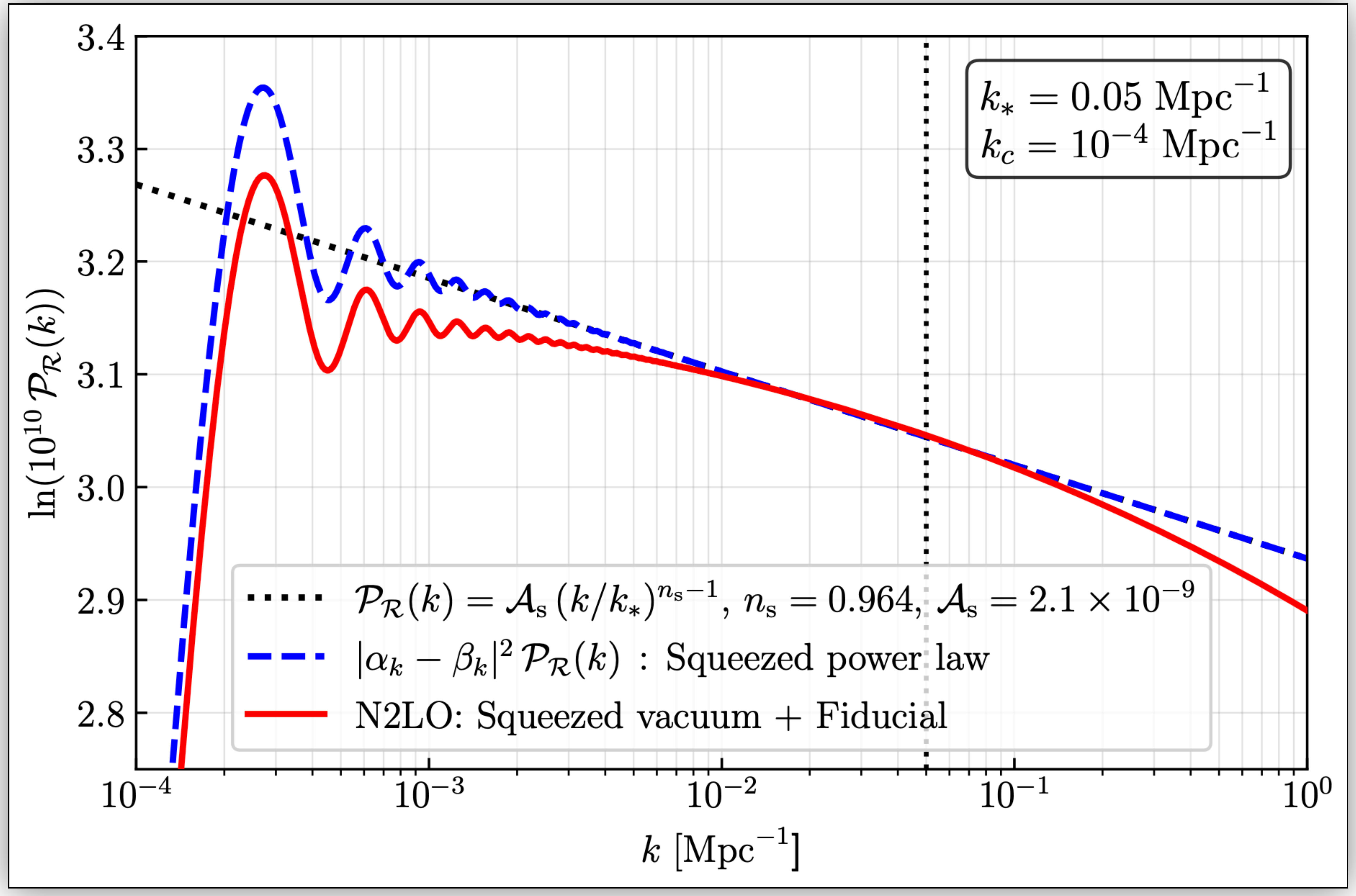
The induced phase δ_k only contains information from the Hubble-flow parameters $\epsilon_{1Hk}, \epsilon_{1Zk}, \epsilon_{1ck}, \ldots$

An illustrative example [Danielsson '02, Martin & Brandenberger '03, Broy '16]

Suppose that initial conditions are set at a finite time $y = y_c$, defining a cutoff scale k_c, such that

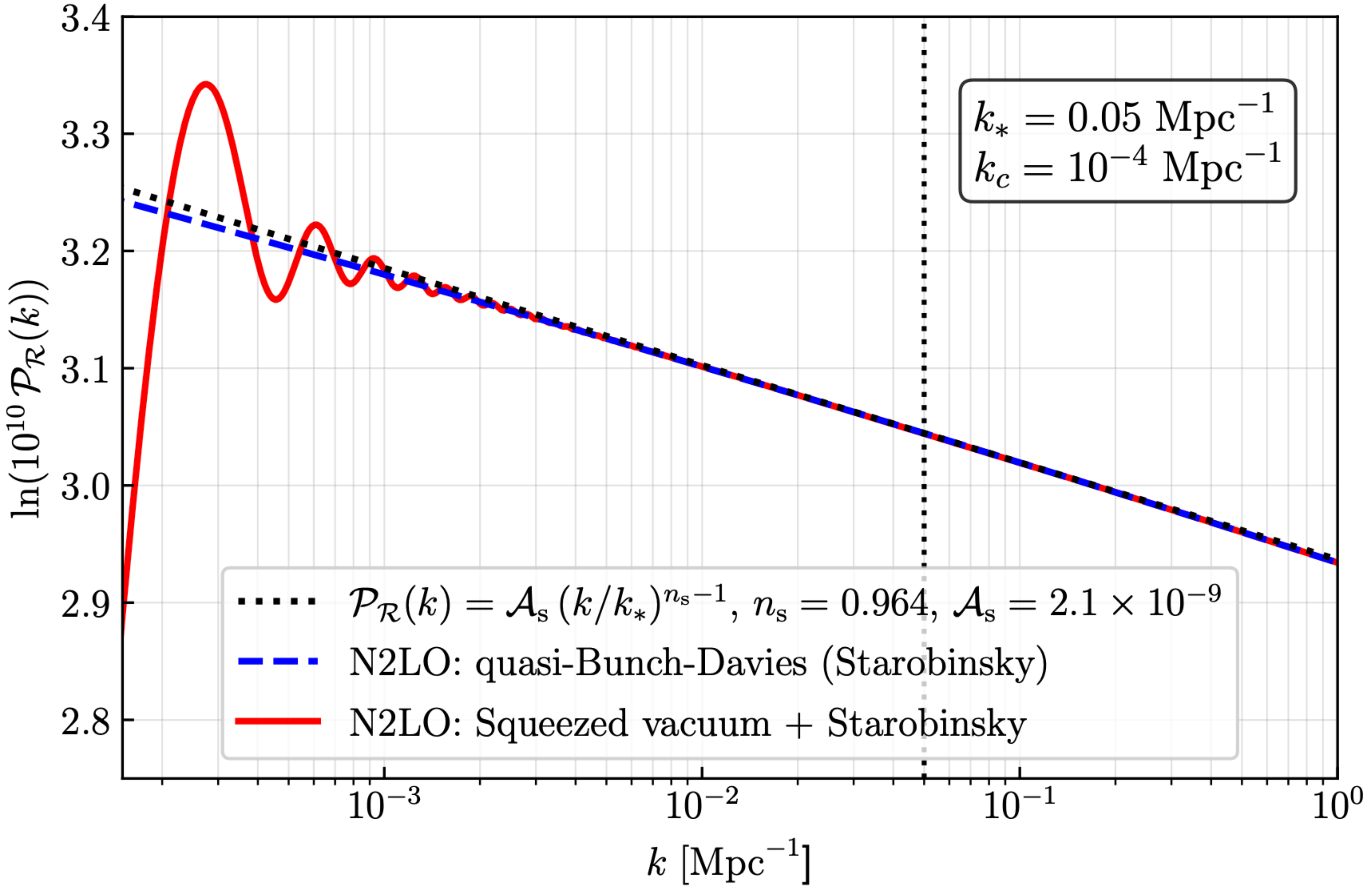
$$\tilde{w}(y_c) = w_{\mathrm{Mink}}(y_c), \; \tilde{w}'(y_c) = w'_{\mathrm{Mink}}(y_c)$$

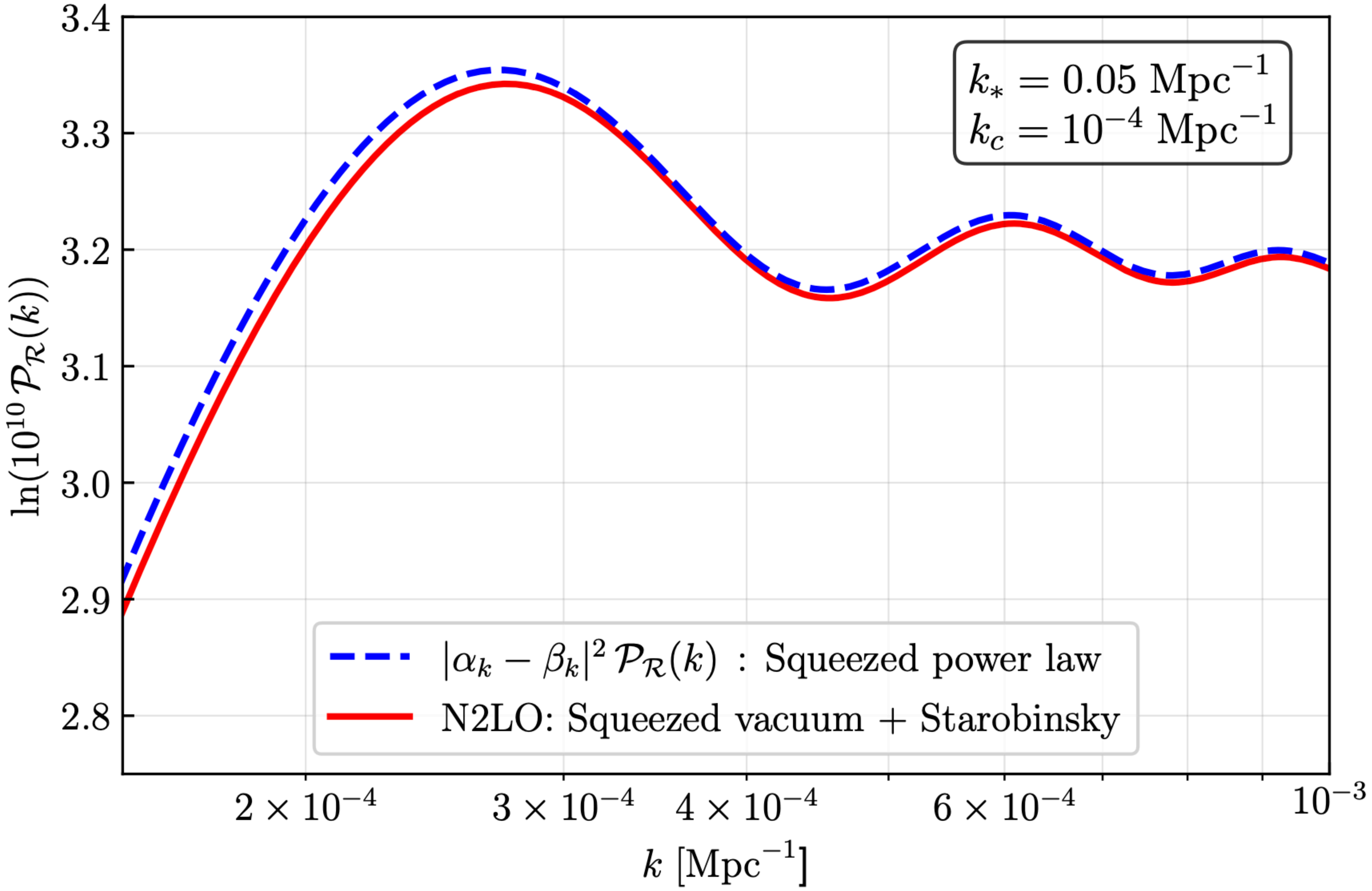
Then, the associated Bogoliubov coefficients are: $\alpha_k = 1 - \frac{1}{2}\left(\frac{k_c}{k}\right)^2 - \mathrm{i}\left(\frac{k_c}{k}\right), \; \beta_k = -\frac{1}{2}\left(\frac{k_c}{k}\right)^2 e^{2\mathrm{i}(k/k_c)}$



Large running,
for illustrative purposes

Slightly more realistic values: Starobinsky [Bianchi & MG, 2024b]





$$\{\epsilon_{1H*} = 0.009, \epsilon_{2H*} = -0.018, \epsilon_{1Z*} = -0.018, \epsilon_{2Z*} = -0.018, \epsilon_{1c*} = \epsilon_{2c*} = 0\}$$

Framework valuable for parameter estimation in future cosmological observations

Conclusions and Future work

- We computed the primordial power spectrum up to N3LO for a wide family of effective theories of inflation. We found non-trivial corrections to Starobinsky inflation.

- **Suggestions received at Loops' 24**:

  ☑ Check results in both frames for Starobinsky (Einstein and Jordan): Results are robust.

  ☑ Extend results for a non-Bunch-Davies vacuum at N2LO.

- **Long term goal (from QG side)**: Explore the emergence of squeezed states from LQG/Spinfoams in Quantum Cosmology and <u>build a bridge</u> towards the QFT regime and phenomenology. Prepare for the precision cosmology era.